Cross Gamma approximations of a portfolio's CVA value

Comparing the performance of "bumping over AAD" for Tape-Based AAD vs Code Generation AAD to compute second-order greeks

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Experiment 3:

- The third and final experiment wanted to test what "bumping over AAD" offers with respect to the cross Gamma approximations of a portfolios CVA value

- Calculating the Cross Gamma Hessian entries w.r.t the points on the Volatility Term structure curves

- How do the approximation methods scale as we increase the computational load on the the respective engines

is defined below:

$$\left.\begin{aligned}\frac{\partial^2 CVA}{\partial\theta_{\sigma_1}\partial\theta_{\sigma_2}} = \frac{CVA\left(\sigma_1^{+\Delta_1}, \sigma_2^{+\Delta_2}\right) + CVA\left(\sigma_1^{-\Delta_1}, \sigma_2^{-\Delta_2}\right)}{4\Delta_1\Delta_2} \\ - \frac{CVA\left(\sigma_1^{+\Delta_1}, \sigma_2^{-\Delta_2}\right) - CVA\left(\sigma_1^{-\Delta_1}, \sigma_2^{+\Delta_2}\right)}{4\Delta_1\Delta_2}\end{aligned}\right\}$$

"Bumping over AAD"

$$\frac{\partial CVA_{\overline{AAD}}}{\partial\theta_\sigma} = \frac{CVA_{\overline{AAD}}(\theta_\sigma + \Delta\theta_\sigma) - CVA_{\overline{AAD}}(\theta_\sigma - \Delta\theta_\sigma)}{2\Delta\theta_\sigma} + \mathcal{O}\left(\Delta\theta_\sigma^2\right)$$

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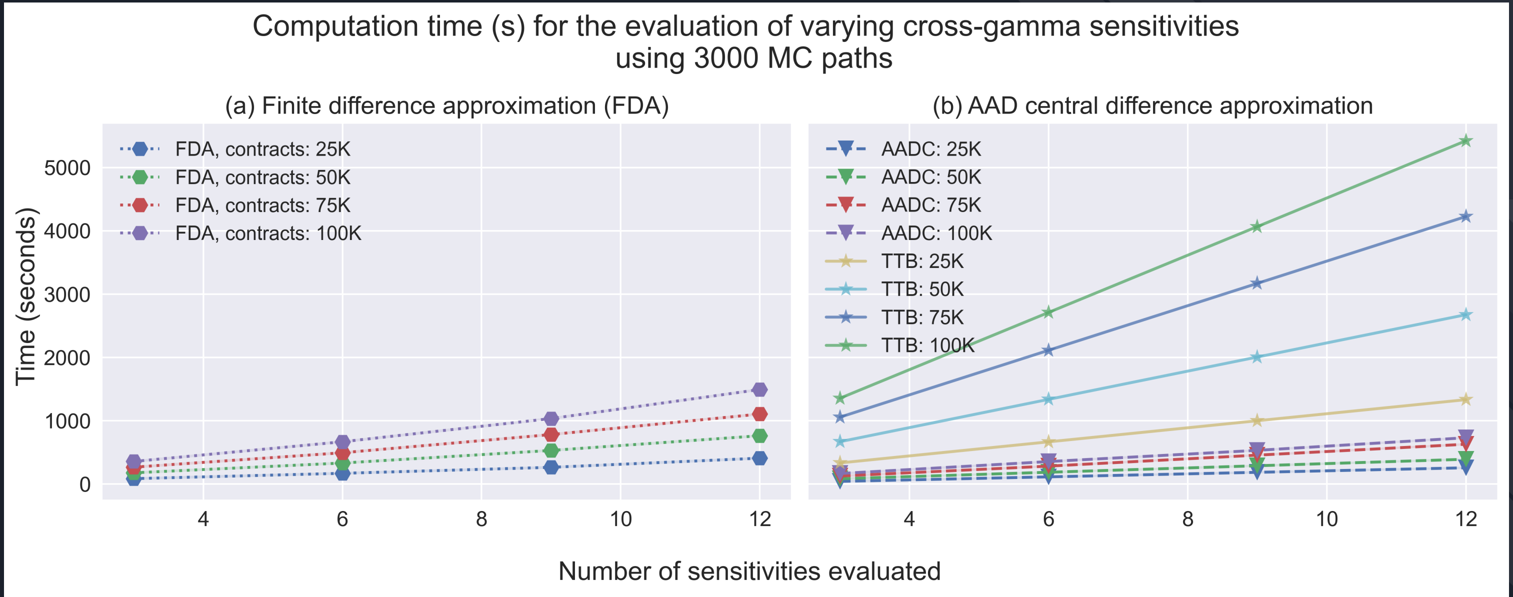
Scaling behaviour of both engines:

Quadratic vs linear scalability



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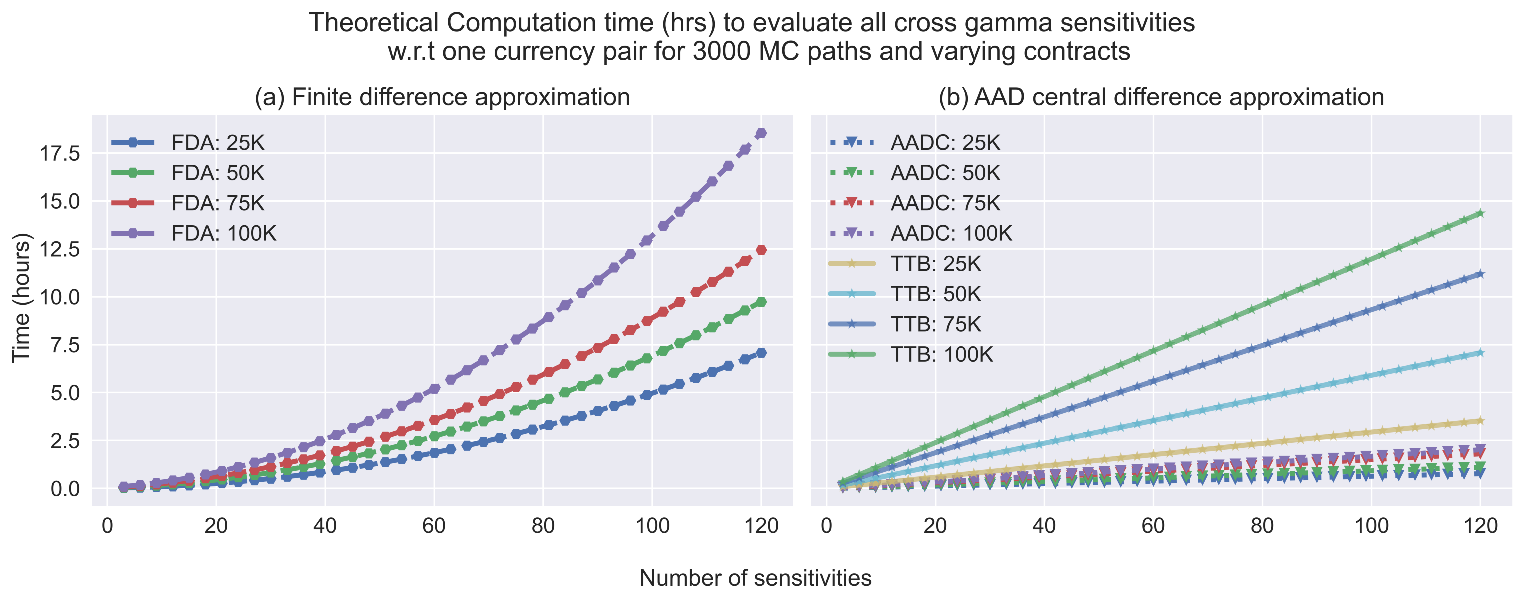

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For all 120 Cross Gamma Hessian entries (between two currencies)

The evaluation of 120 Cross Gamma Hessian **(120 points of the Volatility term structure curves)** entries could then be computed w.r.t to two currencies and

$$\left.\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}\right|_{GBP} = \frac{\left.\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}\right|_{USD} + \left.\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}\right|_{USD}}{2}$$

Method	Number of contracts in portfolio			
	25,000	50,000	75,000	100,000
	Evaluation time (hrs)			
B&R	6.32	8.70	11.11	16.56
B&AADC	0.77	1.10	1.80	2.02
B&TTB	3.52	7.08	11.18	14.35
	Order of improvement attained ($\mathcal{O}$)			
B&AADC	0.96	0.94	0.83	0.96
B&TTB	0.31	0.13	0.046	0.11
	Theoretical speed up			
B&AADC	9.24	8.82	6.90	9.19
B&TTB	2.00	1.37	1.11	1.29

TABLE 4.19: Theoretical time taken, Order of Improvement ($\mathcal{O}$) and computational speed-up realised for the evaluation of all Cross gamma sensitivities (120) with respect to a single currency pair using finite difference approximations vs 'bumping' over AAD. Employing 3000 MC paths

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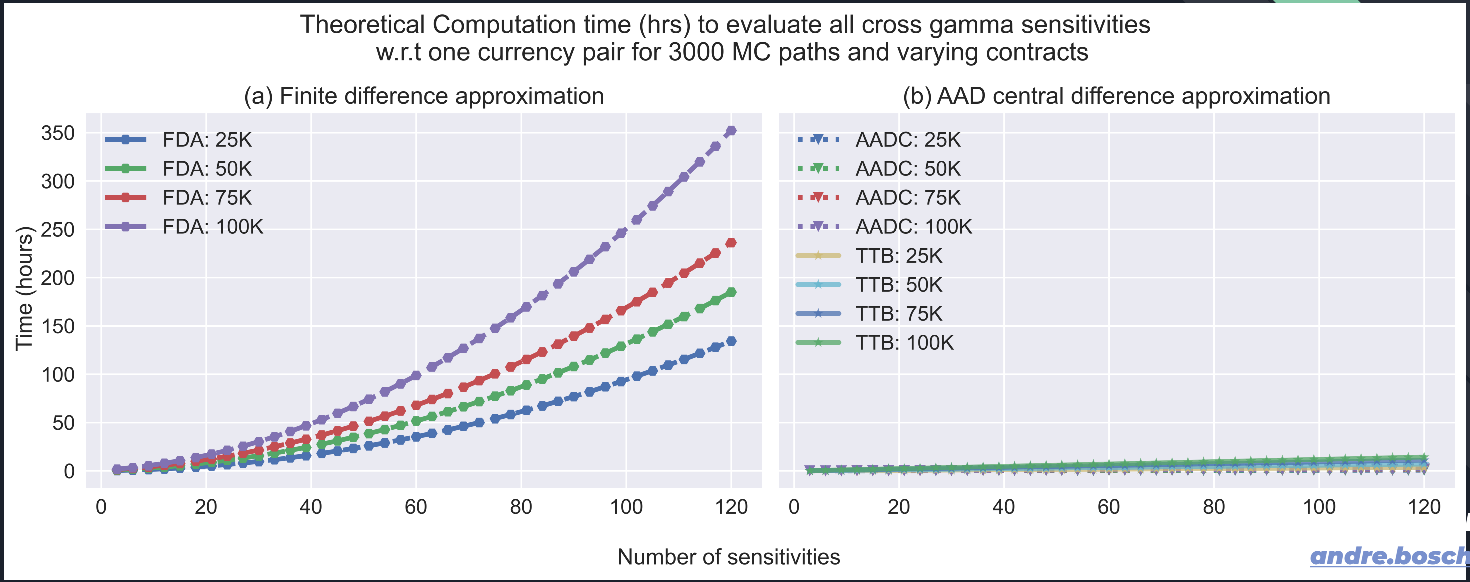
For all 2280 Cross Gamma Hessian entries (between two currencies)

Using "bumping over AAD" does however house an incredibly **scalable advantage**

All first-order adjoints are evaluated w.r.t to the Volatility term structure curves

$$\left.\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}\right|_{GBP} = \frac{\left.\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}\right|_{xCurrencies} + \left.\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}\right|_{xCurrencies}}{2}$$

Essentially producing the remaining 2280 Cross Gamma Hessian Entries by



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For all 2280 Cross Gamma Hessian entries (between two currencies)

Using "bumping over AAD" does however house an incredibly **scalable advantage**
All first-order adjoints are evaluated w.r.t to the Volatility term structure curves

$$\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}_{GBP} = \frac{\frac{\partial CVA_{AAD}}{\partial \theta_\sigma}_{xCurrencies} + \frac{\partial CVA_{AAD}}{\partial \theta_\sigma}_{xCurrencies}}{2}$$

Essentially producing the remaining 2280 Cross Gamma Hessian Entries by producing the original 120 !

Method	Number of contracts in portfolio			
	25,000	50,000	75,000	100,000
	Evaluation time (hrs)			
B&R	120.06	165.23	211.09	314.63
B&AADC	0.77	1.10	1.80	2.02
B&TTB	3.52	7.08	11.18	14.35
	Order of improvement attained ($\mathcal{O}$)			
B&AADC	2.24	2.22	2.11	2.24
B&TTB	1.58	1.41	1.32	1.38
	Theoretical speed up			
B&AADC	175.64	167.64	131.11	174.74
B&TTB	38.11	26.12	21.13	24.54

TABLE 4.21: Theoretical time taken, Order of Improvement ($\mathcal{O}$) and computational speed-up realised for the evaluation of all Cross gamma sensitivities (2240) with respect to a main currency paired with all other currencies using finite difference approximations vs 'bumping' over AAD. Employing 3000 MC paths

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